The Velocity of Current Filaments in Weak Magnetic Fields *

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In the presence of a perpendicular weak magnetic field, current filaments in semiconductors can obtain a transverse drift. In the framework of a collective-coordinate ansatz, an expression for the velocity of the filament is derived for a simple type of extrinsic semiconductors. It turns out that the motion is caused by the coupling of the non-uniform Hall angle to the translational Goldstone mode of the current filament.

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I. Introduction

Recently, current filaments in extrinsic semiconductors at low-temperature avalanche-breakdown have attracted considerable interest, since on the one hand they illustrate the diversity of complex behaviour of self-organized systems, and on the other hand new technical applications appear to be possible [1–5]. In several experiments and theoretical studies a variety of nonlinear phenomena has been found, as e.g. breathing [6], rocking [7], or stochastically firing [8] of current filaments. A rich 'zoo of nonlinear dynamics' [3, 9] in extrinsic semiconductors exists in the presence of an external magnetic field. A recent example is provided by the dynamic Hall-effect [10], where the electric Hall field is treated as a further dynamical variable leading to a Hopf instability. The behaviour of current filaments in an external magnetic field has been investigated experimentally in [11], where it is shown that the nonuniform structures move with transverse velocity. Further, an approximate expression for the velocity is given which turns out to depend strongly on the difference of the mobility of the carriers inside and outside the current filament. In [12], the influence of a magnetic field on the fluctuations near the onset of impact-ionization avalanche-breakdown is studied theoretically in the framework of a linear-mode analysis of the uniform state, and the velocity of a current filament is determined numerically as a function of the applied magnetic field. To my knowledge, however, an exact expression for the value of the velocity of the localized structure has not yet been derived. In the sequel I try to fill this gap in the framework of the collective-coordinate ansatz for the case where the stationary magnetic field is weak.

Let me first recall the simple reasoning given in [11]. The model is based on the Drude single-particle theory of conduction. A current-carrying tube with current density $\boldsymbol{j} \propto \boldsymbol{e}_z$ in an external perpendicular magnetic field $\boldsymbol{B} = B\,\boldsymbol{e}_y$ feels a Lorentz force $\boldsymbol{F} \propto \boldsymbol{j} \times \boldsymbol{B}$ and therefore moves with a (time-dependent) velocity $\boldsymbol{v} \propto -\boldsymbol{e}_x$. In the semiconductor bulk, however, a Hall field $E_{\mathrm{H}}\,\boldsymbol{e}_x$ balances the Lorentz force. Hence the total force acting on each single carrier with charge q vanishes, and a current filament in a perpendicular magnetic field stays at rest, provided that all the carriers have the same mean drift velocity. In the single-particle theory of conduction, the relevant physical quantity associated with a magnetic field is the Hall angle ϕ_{H} between the current density and the electric field. By definition, $\tan(\phi_{\mathrm{H}}) = \mu B$, where μ denotes the mobility of the carriers. One expects a motion of the current filament if ϕ_{H} varies in space, for example resulting from a dependence of the mobility on the carrier density or from a non-uniform magnetic field. In [11], the current filament is treated as a high-conductivity tube consisting of carriers of the density n_1 and with mobility μ_1, embedded in a medium of low-conductivity with carriers of the density n_2 and with mobility μ_2. This implies Lorentz forces of different strengths $\boldsymbol{F}_{1,2} = -q\,\mu_{1,2}\,E_z\,B\,\boldsymbol{e}_x$ acting on the carriers inside and outside the current filament, respectively. These forces are balanced separately by appropriate Hall forces generated by charge densities accumulating at the sample boundaries and at the

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filament walls. Hence, a transverse net force $F_1 - F_2 = q(\mu_2 - \mu_1) B E_z e_x$ acts on the current filament. Assuming a transverse mobility $\bar{\mu}$ of the filament, one concludes for the velocity in e_x-direction

$$v = \bar{\mu}(\mu_2 - \mu_1) B E_z e_x. \quad (1)$$

If $\mu_1 > \mu_2$, for example, the current filament travels in negative e_x-direction. Finally, the authors of [11] assume $\bar{\mu} \equiv \mu_1$, which reproduces quantitatively very well the experimental observations.

It is important to note, however, that the motion of the current filament must not be understood as the drift of carriers, but it rather should be associated with the generation and the capture of carriers from impurity levels at the leading and the trailing part of the filament wall, respectively. Hence, the assumption $\bar{\mu} \equiv \mu_1$ appears to be not justified. The present note is devoted to the generalization and the improvement of (1); in particular, an expression for $\bar{\mu}$ will be derived.

II. The Model

In the following a simple model similar to models in [2] is considered. The basic equations describing transverse structures in the semiconductor are

$$\partial_t n_0 = -\nabla_\perp \cdot (n_0 u_\perp) + f(n_0, 1 - n_0 + \eta \nabla_\perp \cdot E_\perp, w), \quad (2)$$

$$\eta \partial_t E_\perp = -n_0 u_\perp, \quad (3)$$

$$\eta \partial_t E_z = j_z - n_0 u_z, \quad (4)$$

$$u_\perp = \mu(E_\perp + (u \times B)_\perp - n_0^{-1} \nabla_\perp (w n_0)), \quad (5)$$

$$u_z = \mu(E_z + (u \times B)_z), \quad (6)$$

where $\nabla_\perp \equiv (\partial_x, \partial_y)$, $u_\perp \equiv (u_x, u_y)$ etc., and all the quantities appearing in (2)–(6) are dimensionless.

Equation (2) describes the dynamics of charge carriers of density n_0 in a purely extrinsic semiconductor doped with shallow single-level impurities of the normalized density $N \equiv 1$. The generation-recombination (g-r) function f depends on n_0, on the density n_d of occupied impurities and on the mean carrier energy w. The density n_d has already been eliminated in (2) with the help of Poisson's equation $\eta \nabla E = n_0 + n_d - 1$, where η is the (usually small) ratio of the dielectric relaxation time and the characteristic recombination time [2]. Further, the mean carrier energy $w(n_0, E)$ is assumed to depend on the carrier density and on the electric field.

Maxwell's equations (3) and (4) describe the dynamics of the electric field.[1] In the following, $|E_z| \gg |E_\perp|$ is assumed, i.e. terms of $\mathcal{O}(|E_\perp / E_z|)$ will be neglected. For example, $w \approx w(n_0, E_z)$ holds. Another approximation inherent in this model corresponds to the neglect of heat diffusion, disregarding the fact that the heat-diffusion length could play an important role concerning nonuniform structures. In other words, only spatial variations on the Debye length scale are considered. Below, this oversimplification turns out to imply a significant quantitative difference between the numerical values of the analytically derived and the experimentally observed transverse mobility.

The mean carrier velocity u given by (5) and (6) is obtained from an adiabatic elimination based on fast momentum relaxation of the carriers. The diffusion part of the current density is proportional to the gradient of the hydrodynamic pressure $n_0 w$. The (dimensionless) mobility μ is allowed to depend on the state variables in the way $\mu(n_d, E_z)$. This dependence takes account of the difference between ionized and neutral impurity scattering, which are assumed to be the relevant scattering processes at sufficiently low temperatures and may lead to differences in the mobilities and the carrier temperatures inside and outside the current filament.

It must be mentioned that bistability of uniform states does not occur if the nonlinearity origins only from usual single-level g-r kinetics [2], and in order to guarantee the existence of a current filament, the nonlinear dependence of the mobility and the carrier temperature on certain dynamical variables is necessary. However, the following results are not sensitive to the details of these nonlinearities, and it is sufficient to require bistability and the existence of a stable current filament.

III. The Collective-Coordinate Ansatz

Let me first motivate the collective-coordinate ansatz. Consider a current filament without an external magnetic field in a sample with very large lateral extension compared to the largest characteristic intrinsic length. This allows one to assume an effective

[1] The emission of electromagnetic waves by a current filament moving with velocity v is neglected. This effect is at least of order v/c, c being the velocity of light.

translational invariance: a spatial translation of a localized bulk state will generate an equivalent state as long as it does not come too close to the boundaries of the sample. By means of a separation of intrinsic and geometrical length scales, this assumption implies the existence of translational Goldstone modes of the symmetry-breaking current-filament in the bulk.

What is the effect of a weak stationary magnetic field? Intuitively, one expects a small deformation of the stable shape of the current filament, which can be treated in the framework of linear response theory. In the presence of a Goldstone mode, however, the generalized susceptibility has a pole at zero frequency, since the restoring force reacting to a spatial shift of the current filament vanishes. It turns out that this mechanism results in a transverse drift.

For small Hall angles, i.e. $\tan(\phi_H) \approx \phi_H \ll 1$, the drift velocity of the carriers is given by $u_x = u_x^0 - \phi_H u_z^0 + \mathcal{O}(\phi_H^2)$, $u_y = u_y^0$ and $u_z = u_z^0 + \phi_H u_x^0 + \mathcal{O}(\phi_H^2)$, where $\boldsymbol{u}_\perp^0 = \mu(\boldsymbol{E}_\perp - n_0^{-1}\nabla_\perp(w n_0))$ and $u_z^0 = \mu E_z$. The substitution of $u_{x,y}$ into (2) and (3) gives

$$\partial_t n_0 + \nabla_\perp \cdot (n_0 \boldsymbol{u}_\perp^0) - f(n_0, 1 - n_0 + \eta \nabla_\perp \boldsymbol{E}_\perp, w) = \partial_x (n_0 \phi_H \mu E_z), \tag{7}$$

$$\eta \partial_t \boldsymbol{E}_\perp + n_0 \boldsymbol{u}_\perp^0 = (n_0 \phi_H \mu E_z) \boldsymbol{e}_x, \tag{8}$$

where terms of $\mathcal{O}(\phi_H^2)$ are neglected. The current-voltage characteristic of the moving current filament is obtained from the total current $J_{tot} = \int dx\, dy\, j_z$ and from the voltage $U = L_z E_z$, where $j_z = \mu n_0 E_z + n_0 \phi_H u_x^0$ and L_z is the length of the sample in $\boldsymbol{e}_z$-direction. Hence the change of the current-voltage characteristic by a small magnetic field is at least of $\mathcal{O}(\phi_H)$, and the velocity will be computed at fixed E_z.

Let the stationary cylindrical current filament at zero field ($\phi_H = 0$) be denoted by the nonuniform solution $(n_0(\boldsymbol{r} - \boldsymbol{r}_0), \boldsymbol{E}_\perp(\boldsymbol{r} - \boldsymbol{r}_0))$ of (7) and (8), where the collective coordinate $\boldsymbol{r}_0 = (x_0, y_0)$ represents the arbitrary transverse location of the current filament in the sample. Note that the stationary state satisfies $\boldsymbol{u}_\perp^0 \equiv 0$. When a weak magnetic field is switched on, the above mentioned small deformation and the motion of the current filament are described by the ansatz $n_0(\boldsymbol{r} - \boldsymbol{v}t) + \delta n_0$ and $\boldsymbol{E}_\perp(\boldsymbol{r} - \boldsymbol{v}t) + \delta \boldsymbol{E}_\perp$, where δn_0, $\delta \boldsymbol{E}_\perp$ and $\boldsymbol{v}$ are of $\mathcal{O}(\phi_H)$. Insertion into (7) and (8) yields in first order of ϕ_H the linear inhomogeneous partial differential equation

$$\mathbf{L}\begin{pmatrix}\delta n_0 \\ \delta \boldsymbol{E}_\perp\end{pmatrix} \equiv \begin{pmatrix}\partial_t + \nabla_\perp \cdot \mu(\boldsymbol{E}_\perp - \nabla_\perp(n_0 w)') - f' \nabla_\perp \cdot \mu n_0 - \eta \partial_{n_d} f \nabla_\perp \cdot \\ \eta^{-1}\mu(\boldsymbol{E}_\perp - \nabla_\perp(n_0 w)') \quad\quad \partial_t + \eta^{-1} n_0 \mu\end{pmatrix}\begin{pmatrix}\delta n_0 \\ \delta \boldsymbol{E}_\perp\end{pmatrix} = \begin{pmatrix}(\boldsymbol{v} \cdot \nabla_\perp) n_0 + \partial_x (n_0 \phi_H u_z^0) \\ (\boldsymbol{v} \cdot \nabla_\perp) \boldsymbol{E}_\perp + \eta^{-1} n_0 \phi_H u_z^0 \boldsymbol{e}_x\end{pmatrix}, \tag{9}$$

where the prime denotes the derivative d/dn_0 total. The broken translational invariance in the $x-y$ plane implies the existence of two Goldstone modes given by the stationary solutions $\partial_x(n_0, \boldsymbol{E}_\perp)^t$ and $\partial_y(n_0, \boldsymbol{E}_\perp)^t$ of the homogeneous equation associated with (9). The deformation $(\delta n_0, \delta \boldsymbol{E}_\perp)^t$ of the current filament can be determined from (9), provided that the solvability condition on the inhomogeneity is satisfied [13]. This condition (Fredholm alternative) requires that the right-hand-side of (9) vanishes when projected onto the adjoint Goldstone modes being defined as the solutions of $\mathbf{L}^\dagger(\varphi_1(\boldsymbol{r}), \boldsymbol{\varphi}_2(\boldsymbol{r}))^t = 0$, where the projection of $(a_1, \boldsymbol{a}_2)$ onto $(b_1, \boldsymbol{b}_2)$ is defined by $\langle (a_1, \boldsymbol{a}_2) | (b_1, \mathbf{b}_2) \rangle \equiv \int dx\, dy (a_1 b_1 + \boldsymbol{a}_2 \cdot \boldsymbol{b}_2)$. First, the relation $\boldsymbol{\varphi}_2 = \eta \nabla_\perp \varphi_1 - (\eta^2/\mu n_0) \nabla_\perp (\varphi_1 \partial_{n_d} f)$ between φ_1 and $\boldsymbol{\varphi}_2$ can be derived. Introducing the new variable $\varphi \equiv \partial_{n_d} f \varphi_1$ yields finally

$$\left(\eta \Delta + \frac{n_0 f'}{(n_0 w)' \partial_{n_d} f}\right) \varphi = 0, \tag{10}$$

where $\Delta \equiv \nabla_\perp^2$. The calculation of the adjoint Goldstone modes is thus reduced to the Schrödinger-like problem (10) for φ: find the zero-energy wave-function of a particle in two dimensions moving in a potential of the form $V = -n_0 f'/(n_0 w)' \partial_{n_d} f$. Note that in order to guarantee stability of the uniform states associated with positive differential conductivity, $\partial_{n_d} f$ is required to be positive [2]. The current filament in a bistable system can be considered as a cylindrical domain of the locally stable (i.e. $f' < 0$) high carrier-density state, which is connected to the locally stable low carrier-density state outside the current filament by a domain wall crossing a locally unstable region (i.e. $f' > 0$). Therefore, one concludes that the potential V has a minimum at the wall of the current filament and takes negative values only near the wall. Hence, the 'wave function' with zero energy is localized at the filament wall. Due to the cylindrical symmetry of the current filament, the solutions of (10) can be labeled by their multipole character m. The Goldstone modes have dipole character ($m = \pm 1$) and cannot be orthogonal to the space of the adjoint Goldstone modes, hence one concludes that φ also has dipole character. Since the Lorentz force points into $\boldsymbol{e}_x$-direction, it is convenient to choose the symmetry-adapted orthogonal solutions $\varphi^{(x)}$ and $\varphi^{(y)}$ of (10). The function $\varphi^{(x)}$ ('p_x-orbital') is odd in x and even in y, whereas $\varphi^{(y)}$ ('p_y-orbital') is defined reversely. Using the spatial symmetry properties of n_0, $\boldsymbol{E}_\perp$, f' etc., the projection of the inhomogeneity on the right-hand-side of (9) onto $(\varphi_1^{(y)}(\boldsymbol{r}), \boldsymbol{\varphi}_2^{(y)}(\boldsymbol{r}))$ yields $v_y = 0$. The velocity v_x is then obtained from a projection onto $(\varphi_1^{(x)}(\boldsymbol{r}), \boldsymbol{\varphi}_2^{(x)}(\boldsymbol{r}))$, which finally leads to

$$v_x = \frac{\eta E_z}{C} \int \varphi^{(x)} \partial_x \phi_{\mathrm{H}} \, \mathrm{d}x \, \mathrm{d}y, \qquad (11)$$

where $C \equiv -\langle(\varphi_1^{(x)}(\boldsymbol{r}), \boldsymbol{\varphi}_2^{(x)}(\boldsymbol{r})) | \partial_x (n_0, \boldsymbol{E}_\perp)\rangle$.

IV. Discussion

The result (11) simply states that a motion of the nonuniform structure is caused by the coupling of a spatial gradient of the Hall angle to the Goldstone mode, i.e. a current filament moves if the Hall angle is spatially nonuniform.

In order to recover (1) with the help of (11), consider the case of a uniform magnetic field, i.e. $\partial_x B \equiv 0$, and the mobility of the form $\mu = \mu_1 + (\mu_2 - \mu_1) n_{\mathrm{d}}$. This immediately yields (1) with the transverse mobility

$$\bar{\mu} = \eta C^{-1} \int \mathrm{d}x \, \mathrm{d}y \, \varphi^{(x)} \partial_x n_{\mathrm{d}}. \qquad (12)$$

The result (11) also provides a possibility to control the current filament by an appropriate nonuniform magnetic field. For example, if the characteristic length of spatial variations of $B(x)$ is large compared to the filament diameter l, (11) yields $\dot{x}_0 \equiv v_x \propto l \, \partial_{x_0}^2 B(x_0)$; here, $\varphi^{(x)}$ has been approximated by delta functions centered at the wall of the current filament. The current filament can be pinned in an inhomogeneous magnetic field, or even moved by a slow temporal variation of the field. Another application of (11) is the inclusion of the magnetic long-range interaction between current filaments.

Since $\bar{\mu} = \mathcal{O}(\eta)$ is small, the theoretical result (12) does not fit quantitatively well the experimental results reported in [11]. Nevertheless, I believe that the method displayed in this note is the appropriate one in order to determine the correct mobility $\bar{\mu}$, but that the model given by (2)–(6) is not the appropriate one to describe the experiments of [11]. The factor η originates from the determination of the adjoint Goldstone modes, where $\sqrt{\eta}$ is associated with the characteristic length scale of the model. In a more appropriate model, the inclusion of heat diffusion introducing an additional length scale should lead to a mobility $\bar{\mu}$ which is in better agreement with the experimental observations.

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